

# IMPORTANCE MEASURES IN RELIABILITY AND MAINTENANCE

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# OUTLINE



## Introduction

The role of IM in reliability and maintenance

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## IM history and classes

Structural, reliability, lifetime, and cost-based IM

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## System state

Binary, multi-state, continuous IM

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## Group of components

IM for group of components, optimization approach

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## Conclusions

# Importance measures

## Brinbaum

“In a system whose performance depends on the performance of its components, some of these components may play a more important part than others.”

Importance measures (IM) are used to estimate the relative criticality of systems' components.

Reliability terminology: IMs measure the influence of components' reliability on the overall system reliability.

# The role of importance measures

## **System design**

To determine the appropriate reliability of each of the system's components.

## **System improvement**

To identify the weakest areas of a system whose reliability should be improved.

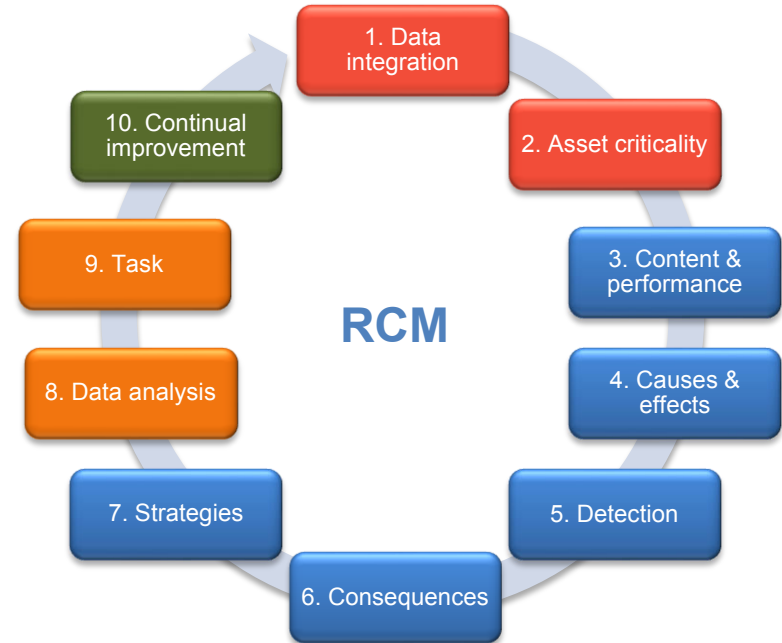
## **System maintenance**

To identify the components that need special attention in the maintenance process.

# The role of IM in Reliability Centered Maintenance (RCM)

In RCM analysis of component criticality is one of pre-work internal sub-process\*.

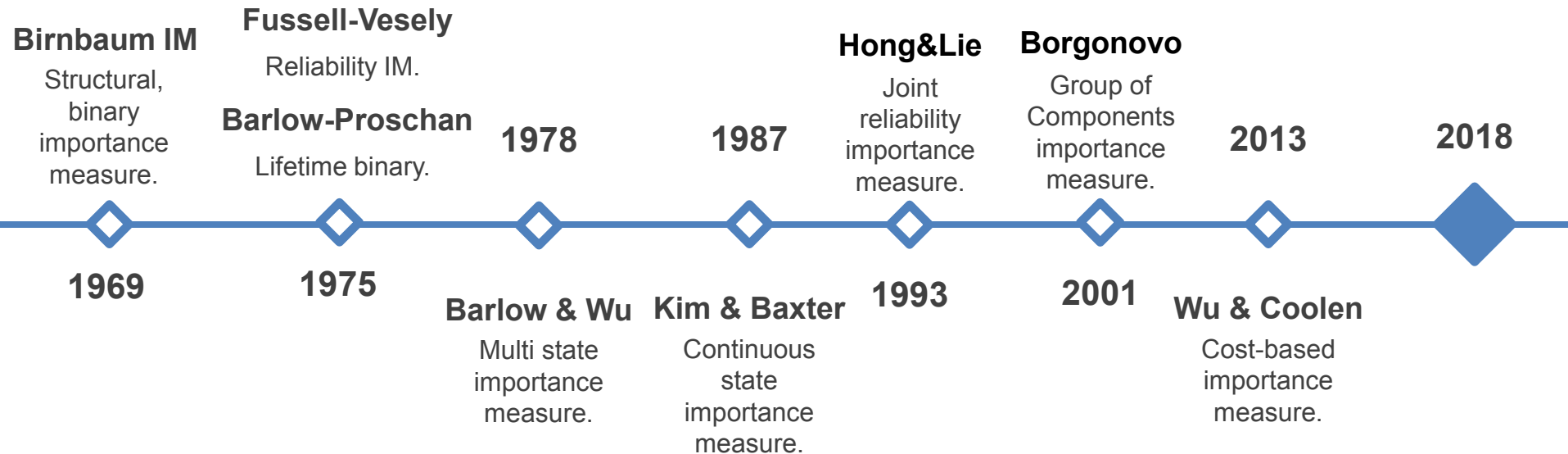
RCM tend to identify the components that are critical for the system reliability and to direct maintenance efforts towards these components\*\*.



\*Sifonte, J. R., & Reyes-Picknell, J. V. (2017). *Reliability Centered Maintenance—Reengineered: Practical Optimization of the RCM Process with RCM-R®*. Taylor & Francis Group.

\*\*Zio E. (2009) *Reliability engineering: Old problems and new challenges*. *Reliability Engineering and System Safety* 94, 125–141

# IM development



# European safety and reliability association (ESRA)

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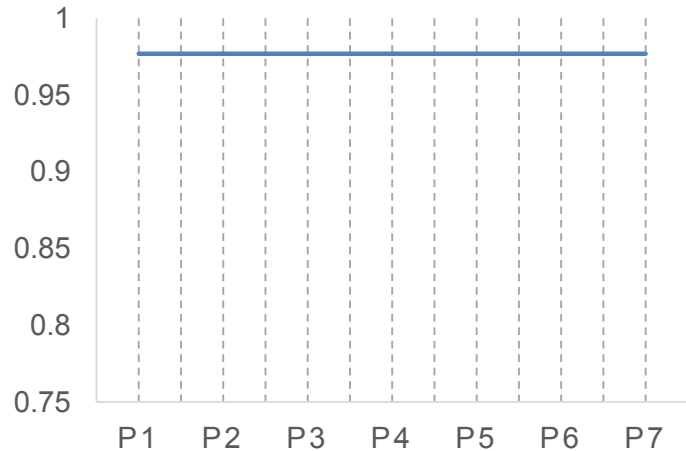
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# IMPORTANCE MEASURES CLASSES AND TYPES



# Birnbaum Importance Measure

$$I^B(i | t) = h(1_i, p(t)) - h(0_i, p(t))$$



— base

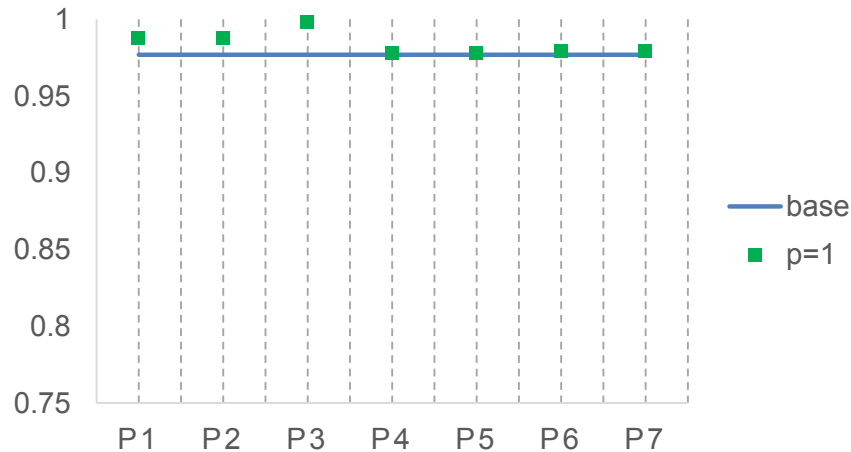
## Structure importance measure

Introduced by Birnbaum (1969) in order to analyze criticality of components in coherent systems.

Structural IM that measures the difference between the system's reliability when the component  $i$  is in perfect functioning state and the system's reliability when the component  $i$  is complete failure state.

# Birnbaum Importance Measure

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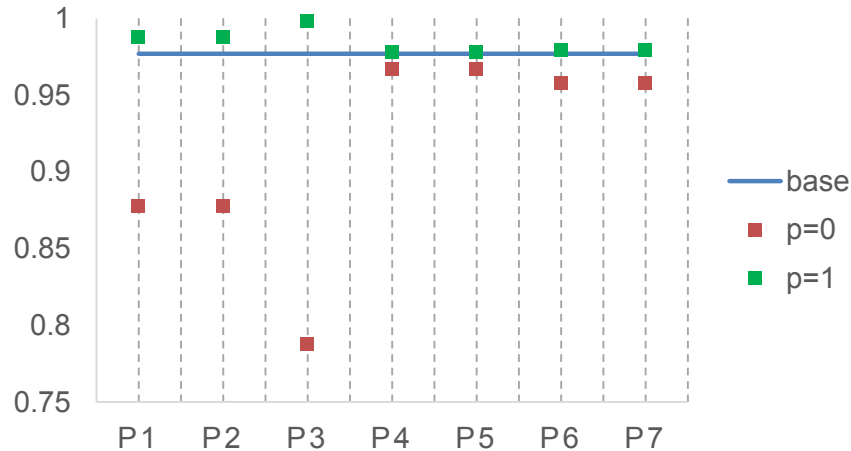
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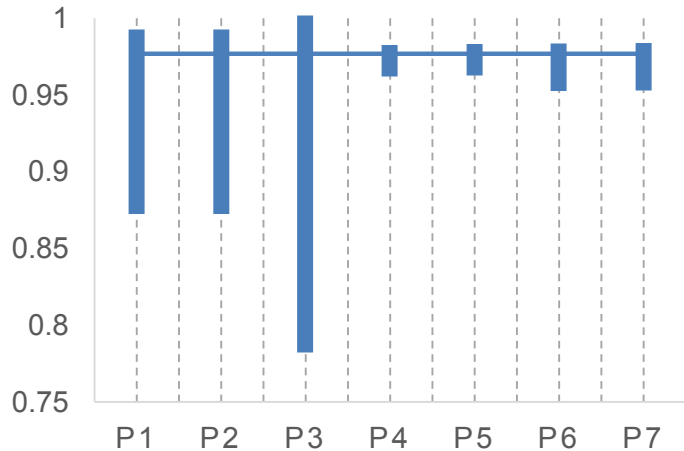
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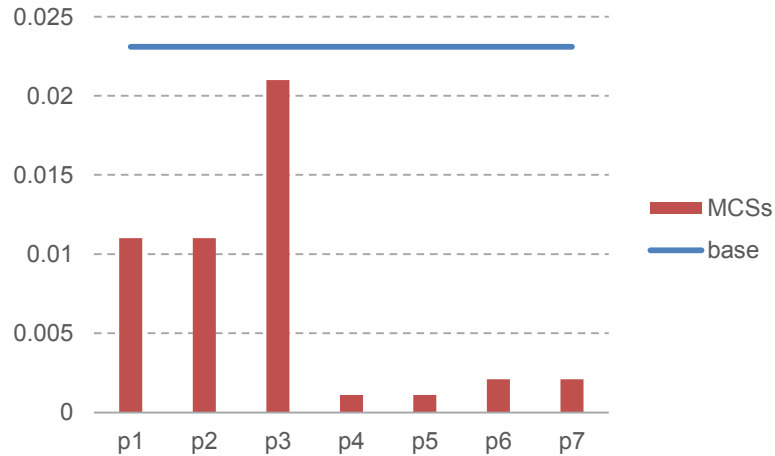
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# Fussell–Vesely Importance Measure

$$I^{FV}(i|t) \approx \sum_{j=1}^{m_i} Q_i^j(t)$$



## Reliability importance measure

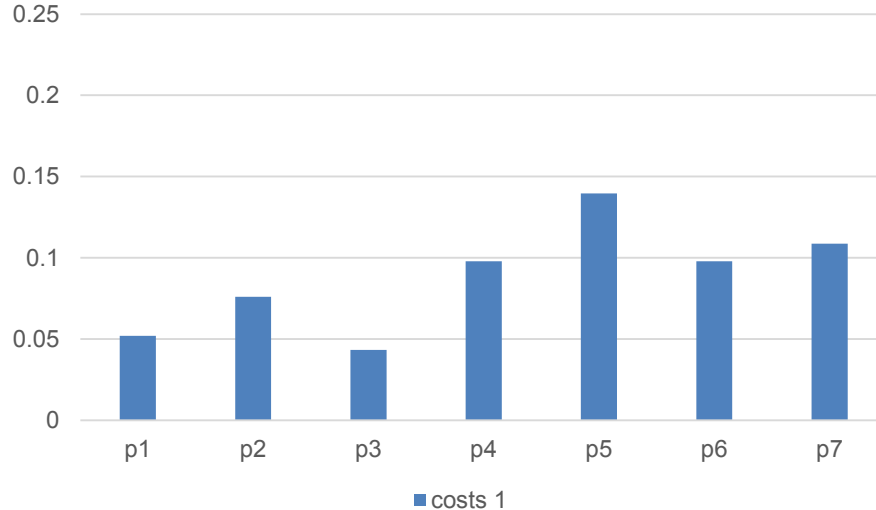
Introduced by Fussell and Vesely (1985).

Defined through minimal cut sets (MCS).

MCS are combinations of components whose simultaneous failure cause the system failure.

## Cost-based Importance Measure

$$I_i^{CBCI}(t) = -\frac{\partial C_i(t)}{\partial R_i}$$



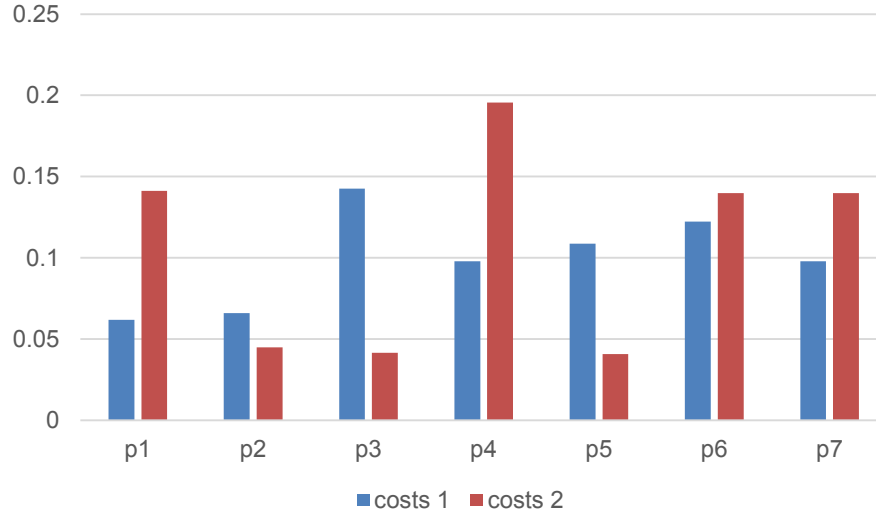
## Cost-based importance measure

Introduced by Wu and Coolen (2013).

Considers the influence of cost performance improvement of some component on system reliability.

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# Multi-state components and systems

Components and the system have several discrete degradation states between perfect functioning and complete failure.

## Some examples

Each k-out-of-n system.

Solar generators that consist of many independent solar modules and whose state depends on the number of functioning solar modules.

Capacity of the production system with any configuration (series, parallel, series-parallel or network) depends on the states of the machines.

Systems that contain switching components have different states depending on switching components failure modes: fail to close at closing command or fail to open at opening command.



# Multi-state components and systems

Components and the system have several discrete degradation states between perfect functioning and complete failure.

## Approaches

Adoptions of the Birnbaum, Fussell-Vesely, RAW and RRW importance measures to multi-state systems.

Threshold approach: let  $\alpha, \alpha \in (0, 1)$  be threshold state and  $s$  a component's state. Then, states  $s < \alpha$  represent the failure states and  $s \geq \alpha$  represent the operating states.

Criticality determination through minimal cut sets and minimal path sets.

# Multi-state IM defined through MCSs

$$s = \min_{C_j \in C} \max_{i \in C_j} s_i$$

$K = \{1, 2, \dots, n\}$  set of system components

$C = \{C_1, \dots, C_k\}$  set of MCS

$S = \{0, 1, \dots, m-1\}$  set of system states,  
0: failure,  
 $m-1$ : perfect functioning

$s_i \in S, i \in K$  state of the component  $i$

$s \in S$  state of the system

The system state is equal to the state of the "best" component in the "worst" MCS.

# Continuous- state importance measures

Components and the system have several continuous degradation states between perfect functioning and complete failure.

## Some examples

Boilers in coal-fired power station can produce megawatts less than their full capacities.

Valves in internal combustion engine degradation can wear after a large number of engine cycles.

The performance of an automobile tire degrades continuously as the tread wears.

Nuclear waste repository is comprised of passive components that function and degrade in a continuous fashion.

Capacities of the machines in production system can decrease continuously with time.

# Continuous- state importance measures

Components and the system have several continuous degradation states between perfect functioning and complete failure.

## Approaches

Adoptions of traditional importance measures to continuous-state systems.

Threshold approaches that reduce continuous state to binary state.

Partitioning approaches that reduce continuous state to multi-state.

Criticality determination through minimal cut sets and minimal path sets.

## Continuous-state IM defined through MCSs

$$s = \min_{C_j \in C} \max_{i \in C_j} s_i$$

$K = \{1, 2, \dots, n\}$  set of system components

$C = \{C_1, \dots, C_k\}$  set of MCS

$s_i \in [0, 1], i \in K$  state of the component  $i$

$s \in [0, 1]$  state of the system

States: 0 = failure, 1 = perfect functioning

The system state is equal to the state of the "best" component in the "worst" MCS.

# Group importance measures

## **Open issue on importance measures**

Majority of IM rank only individual components according to calculated values of chosen measure.

They are not directly applicable to combinations or groups of components.

## Joint Reliability IM

$JRI(i, j)$

$$JRI(i, j) = \frac{\partial^2 h(\mathbf{p})}{\partial p_i \partial p_j}$$

$p_i(t)$  component  $i$  reliability

$h(p(t))$  system reliability

### Pair reliability importance measure

Introduced by Hong and Lie (1993) in order to measure joint impact of pairs of components on system reliability.

# Optimization approach

LabOI “Jovan Petrić”, FON

$$\max s = \min_{C_j \in C} \max_{i \in C_j} s_i$$

s.t.

$$\sum_{i \in K} c_i(s_i) \leq b$$

$$s_i \in \{0, 1, \dots, m-1\} \text{ or } s_i \in [0, 1]$$

Criticality of group of multi-state or continuous-state components respecting the components improvement costs.



# CONCLUSIONS



## The role of IM

Identification of critical components is important phase in system design, improvement and maintenance processes.

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## Multi and continuous states

Systems and its components often have many degradation states between perfect functioning and total failure. This should be included in components criticality analysis.

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## Costs of components improvement

Besides system structure and components reliability, cost of component improvement can influence on component criticality.

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## Group of components

Joint influence of group of components give additional information about component criticality. Determination of group of critical components can be defined as optimization problem.

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