IMPORTANCE MEASURES IN RELIABILITY AND MAINTENANCE

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Abstract: *Identification of critical components is one of the most demanding tasks in designing complex systems and determination of priorities in maintenance planning process. In reliability theory, importance measures are used to estimate the relative criticality of systems' componentsi.e. to rank the components according to the impact of their reliability on the overall system reliability.The concept of importance measures was originally introduced by Birnbaum in the 1960s after which many of different approaches and measures have been developed continuously. The aim of this paper is to present general concept, historicaldevelopment, types and applications fields of importance measures.Special attention will be focused on the main groups of importance measures, depending on components data used in analysis: structure, reliability, lifetime, and cost related importance measures. Further, importance measures features will be analysed in relation to the failure states of system and its components: binary, with perfect functioning and complete failure states; multi-state, with several discrete degradation states between functionality and failure; and continuous, with continuous degradation states. Finally, some open issues and disadvantages of importance measures will be presented. One of the disadvantages of the existing importance measures is the inability to identify simultaneously the set of a given cardinality of critical components. A new optimization approach based on set covering problem, that overcome this shortcoming, will be presented in the last section of the paper.*

Key words: *importance measures, component criticality, reliability, maintenance*

1. INTRODUCTION

Asset management includes the understanding of the range of assets that are used in the business, the roles of assets and the relative criticality of particular assets (Hastings, 2010). The focus of this paper is on criticality analysis. Understanding and assessing component criticality is also one of importance phases in many maintenance strategies. In Reliability centred maintenance (RCM), for example, analysis of component criticality is one of pre-work internal sub-process(Sifonte & Reyes-Picknell, 2017). In preventive maintenance (PM), Wu et al. (2016) propose that,when a component in the system fails, PM should is carried out on other critical components while the system is already unavailable. In reliability theory, components criticality is analysed using importance measures. The first importance measure was introduced by Birnbaum in the late sixties and since then the concept is constantly developing.

The aim of this paper is to give an overview of the most used important measures (IM). The paper consists of five section. Section 2 is devoted to history of IM development and their classes regarding data available for the analysis. Section 3 is related to the importance measures types according to components and system states. Section 4 refers to the one of the open issue in IM area - IM for groups of components and gives some of the existing solution. Conclusions are given in Section 5.

2. IMPORTANCE MEASURES HISTORY AND CLASSES

The choice of IM depends on the aim pursued by the analysis but also on the available data. If only the structure of the system is known, Structural IM are used. The data about components reliabilities enable the usage of Reliability IM. Lifetime IM includes lifetime of the components in the criticality analysis. Finally, if costs of components improvement or maintenance are significantly high, cost based IM should be applied. There are IMs that introduce uncertainty of data in components criticality analysis but they are beyond the scope of this paper.

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2.1. Structural importance measures

The first importance measure was introduced by Birnbaum (1969) in order to analyse criticality of components in coherent systems.Birnbaum importance measure (BI measure) of some component i is structural importance measure because it depends only on structure of the system and the reliability of the remaining system's components and does not depend on the actual reliability of component i. The BI measure expresses the probability that the component i is critical to the system at the moment t. Let the observed system consists of n components and let p(t)=(p1(t),..., pn(t)) be a vector of components reliabilities. Then, BI measure can be defined as (Birnbaum, 1969):

$$I^B(i\,|\,t) = \frac{\partial h(p(t))}{\partial p_i(t)} \quad (1)$$

where $h(p(t))$ is the reliability of the system. A large $I^B(i\,|\,t)$ value indicates that small changes in the reliability of components cause large changes in system reliability. Consequently, the higher the $I^B(i\,|\,t)$ value, the component is more critical.

Considering that, based on the decomposition theorem, the following holds (Krčevinac, Čangalović, Kovačević-Vujčić, Martić, & Vujošević, 2004):

$$h(p(t)) = p_i(t)\cdot h(1_i, p(t)) + \left(1 - p_i(t)\right)\cdot h(0_i, p(t)) = p_i(t)\cdot\left[h(1_i, p(t)) - h(0_i, p(t))\right] - h(0, p(t))$$

and BI measure (1) is then(Hoyland & Rausand, 1994):

$$I^B(i\,|\,t) = h(1_i, p(t)) - h(0_i, p(t)) \quad (2)$$

Hence, the BI measure of a component i represents the difference between the system's reliability when the component i is in perfect functioning state and the system's reliability when the component iis complete failure state.

Over time, the Birnbaum measure (BI) was expanded in different ways and adjusted to different characteristics of the observed system: by introducing costs in analysis(Gao, Baranady, & Markeset, 2010), (Wu & Coolen, 2013), (Débieux, Sivanthi, & Pignolet, 2017); by expanding to non-coherent systems (Andrews & Beeson, 2003), (Vaurio, 2016); by introducing uncertainty in analysis(Baraldi, Compare, & Zio, 2013) (Giuseppe, Maria, & La Fata, 2016); by applying to multi-state and continuous systems (Ramirez-Marquez & Coit, Composite importance measures for multi-state systems with multi-state components, 2005) (Dui, Si, Cui, Cai, & Sun, 2014) etc. Therefore, s good understanding of BI makes it easier to understand the large number of other importance measures.

Among the measures that have been inspired by BI measure, the most widely used are Risk Achievement Worth(RAW) and Risk Reduction Worth, (RRW) (Cheok, Parry, & Sherry, 1998) and its reliability variants: Reliability Achievement Worth(RWA) and Reliability Reduction Worth (RWR) (Espitrity, Coit, & Prakash, 2007). RAW and RRW of component imeasure the impact of component total failure (pi(t)=0) and perfect functioning (pi(t)=1) on system unreliability, respectively. This impact is calculated using following equations:

$$I^{RAW}(i\,|\,t) = \frac{1 - h(0_i, p(t))}{1 - h(p(t))}, \quad I^{RRW}(i\,|\,t) = \frac{1 - h(p(t)}{1 - h(1_i, p(t))} \quad (3)$$

Analogously, RWR and RWA of component i measure the impact of component total failure (pi(t)=0) and perfect functioning (pi(t)=1) on system reliability, which is expressed as:

$$I^{RRW}(i\,|\,t) = \frac{h(p(t)}{h(0_i, p(t))}, \quad I^{RAW}(i\,|\,t) = \frac{h(1_i, p(t))}{h(p(t))} \quad (4)$$

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2.2. Reliability importance measures

Fussell–Vesely importance measure (FVIM) is reliability IM that differs conceptually from BI measure because it is defined through minimal cut sets (MCS) and/or Minimal path sets (MPS)(Kuo & Zhu, 2012). Here, MCS formulation of FVIM will be provided. Cut set is a set of events that together cause the top undesired event to occur. Minimal cut set is a cut set reduced to a minimum number of events that cause the top undesired event to occur (Ericson II, 1999).

FVIM is based on the fact that the system will fail at the moment t if at least one MCS is being realized, which means that all the components of this MCS should fail at the moment t. The component contributes to the system failure when the MCS that contains this component is realized. FVMI of component ican be calculated as (Vesely & Davis, 1985):

$$I^{FV}(i\,|\,t) \approx \frac{1-\prod_{j=1}^{m_i}(1-\breve{Q}_i^j(t))}{Q(t)} \approx \frac{\sum_{j=1}^{m_i}\breve{Q}_i^j(t)}{Q(t)} \tag{5}$$

where $\breve{Q}_i^j(t)$ is the probability of MCSj that contains componenti and $Q(t)$ is the probability of system failure. Hence, FVIM of the component i represents the ratio of the sum of the MCS that contain component i probabilities and probability of system failure. Since $Q(t)$ does not influence on components ranking, components can be ranked based on the values of denominators in (5).

The FVIM is also called Diagnostic Importance Measure (Dutuit & Rauzy, 2015) and it has been modified in various ways, but significantly less than the BI measurement. FVIM was modified in order to measure: the importance of components in noncoherent systems (Vaurio, 2016), the importance of the selected group of components (Cheok, Parry, & Sherry, 1998), the importance in dynamic reliability analysis (Tyrväinen, 2013) and the importance of components with fuzzy reliability (Wang, Zhang, & Chen, 2013). FVIM was combined with Monte Carlo simulation for uncertainty analysis (da Silva Borges, Lava, Guimarães, & de Lourdes Moreira, 2015).Kvassay et al. (2015)applied direct partial logic derivatives in order to identified only MCSs that contain specified component and and thus made it easier to calculate FVIM.

Besides FVIM based importance measures, Criticality IM (CR) is a reliability IM that is widely used in components criticality analysis. CR of component i represents an extension of BI measure with a values of actual probability of component i (Van der Borst & Schoonakker, 2001):

$$I^{CR}(i\,|\,t) = \frac{I^{B}(i\,|\,t)}{h(p(t))}\,p_i(t) \tag{6}$$

where $I^{B}(i\,|\,t)$ is calculated using equation (1).

2.3. Lifetime importance measures

Lifetime IMs consider the structure of the system, the probability and lifetime of the components. The primus representative of lifetime IM is Barlow- Proschan (BP)IM, which takes into account the lifetime of the component for a longer time period and thus provides a more general assessment of the importance of the component (Kuo & Zhu, 2012). BP measure of component i represents the probability that the component i is critical in the infinitely long operating time of the system t→∞(Barlow & Proschan, 1996)(Barlow & Proschan, 1975):

$$I^{BP}(i\,|\,t) = \int_0^{\infty}\left(h\left(1_i, p(t)\right) - h\left(0_i, p(t)\right)\right)\,dp_i(t) \tag{7}$$

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It should be noted that all the above measures have lifetime variants which can be found in (Amrutkar & Kamalja, 2017) and (Kuo & Zhu, 2012).

2.4. Cost based importance measures

Since the improvement of component state as well as the maintenance process involve certain costs, impact of this cost can also be included in component criticality analysis. The number of cost based IMs is still small and this is one of developing issues in IM area. Here, two cost based IMs will be presented.

Cost-effective IM (CEIM) of component i is defined as a combination of IM concept and costs caused by failures(Gupta, Bachttacharya, Barabady, & Kumar, 2013):

$$I_i^{CEIM}(t) = \frac{I_i^{GI}(t)}{C_{f,i}} \quad (8)$$

where I_i^{GI} represents the importance of the component i in the moment tand $C_{f,i}$ represents cost factor of the component i. I_i^{GI} is calculated as a ratio of change in the unreliability of a system caused by a change in unreliability of the component i and the base system unreliability. Cost factor, $C_{f,i}$ represents the ratio of total expected cost of all components failures and expected cost of component i failure.

$$I_i^{GI}(t) = \frac{\Delta_i Q(t)}{Q(t)}, \quad C_{f,i} = \frac{\sum_{i=1}^{n} E(C_i)}{E(C_i)} \quad (9)$$

Cost-based IM (CBCI) considers the influence of maintenance cost of some component on system reliability(Wu & Coolen, 2013). CBCI of componentiis defined as:

$$I_i^{CBCI}(t) = -\frac{\partial C_i(t)}{\partial R_i} \quad (10)$$

where $\partial C_i(t)$ and ∂R_i represent total costs and system reliability increases caused by maintenance of component i, respectively.

3. IMPORTANCE MEASURES ACCORDING TO COMPONENTS AND SYSTEM STATES

All the IMs listed in the previous section refer to the system and components with binary states: perfect functioning and complete failure states. However, there are many systems whose components have several discrete degradation states or continuous degradation states between perfect functioning and complete failure. Consequently, such system can also have states of functioning in some degraded mode. Developing IMs for those system is one of the most challenging area in reliability theory (Yingkui & Jing, 2012). There are two groups of such IMs, depending on whether the degraded states of the components are discrete or continuous: multi-state IM and continuous IM.

3.1. Multi-state importance measures

There are many systems in practice that can be considered multi-state, such that the system and its components can have several discrete degradation states between perfect functionality and total failure. The most general case is each k-out-of-n system where system is in functional state if k or more components are available but the system performance level depends onthe number of available components (Lisnianski, Frenkel, & Ding, 2010).Solar generators consists of many independent solar

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modules and its states depends on the number of functioning solar modules (Li & Zio, 2012).The similar analysis of the other power systems as multi-state can be found in the literature: wind turbines (Wu, et al., 2009), coal power generating units (Lisnianski , Elmakias, Laredo, & Haim, 2012), gas fuelled cogenerated power plant(Reshid & Abd Majid, 2011) etc. Systems that contain switching components have different states depending on switching components failure modes: fail to close at closing command or fail to open at opening command (Levitin, 2002). Capacity of the production system with any configuration (.series, parallel, series-parallel or network) depends on the states of the machines (Nourelfath, Fitouhi, & Machani, 2010).

Multi-state systems can be defined through MCSs. Let $K=\{1,2,...,n\}$ be a set of multi-state components and let $C=\{C_1,...,C_m\}$ be a set of MCSs. The system and components can be in one of the m states $S=\{0,1,...,m-1\}$, where the 0 corresponds to the failure state and m-1 corresponds to the state of perfect functionality. The state of the system is denoted by s and the states of the components are denoted by $s_i, i \in K$. The state of the system can be determined as (Lisnianski, Frenkel, & Ding, 2010):

$$s=\min_{C_j \in C}\max_{i \in C_j} s_i . \qquad (11)$$

The equation (11) implies that the system state is equal to the "best" component in the "worst" MCS (Barlow & Wu, 1978).

The first analysis of multi state systems was conducted byBarlow and Wu (1978) but the first IM for multi-state systemsappeared ten years later. One of the first IMs developed for multi-state systems can be found in (Bossche, 1987). The majority of multi-state IM are developed as adoption of the existing binary IM(Vujošević, Makajić-Nikolić, & Pavlović, 2017). Levitin and Lisnianski (1999) made one of the first extension of binary IM to multi-state systems. Later, Zio and Podofillini(2003) adopted Birnbaum, Fussell-Vesely, RAW and RRW to multi-state systems. Xiahou, Liu, and Jiang (2018) extended BI, RAW, RRW, and FV importance measures to multi-state systems with uncertainty of state assignment. Several new IM for multi-state were developed, one of which relates to multi state systems that includes costs in components criticality determination (Ramirez-Marquez & Coit, 2007). Partial logic derivatives used by Kvassay et al. (2015) was also used for determinations of criticality of multi-state components (Zaitseva, 2012), (Kvassay, Zaitseva, & Levashenko, 2015).

3.2. Continuous-state importance measures

Continuous-state system and components can have continuous degradation states between perfect functionality and total failure. In practice, there are many examples of such systems and components. Boilers in coal-fired power station can produce megavolts less than their full capacities (Baxter & Kim, 1986). Valves in internal combustion engine degradation can wear after a large number of engine cycles (Yang & Xue, 1996). The performance of an automobile tire degrades continuously as the tread wears (Russell & Kailash, 1999). Nuclear waste repository is comprised of passive components that function and degrade in a continuousfashion(Eisenberg & Sagar, 2000). Capacities of the machines in production system can decrease continuously with time (Kuang, Dai, & Zhao, 2013).

The state of the continuous-state system can be determined through MCSs using equation (11) where system state $s \in [0,1]$ and components states $s_i \in [0,1], i \in K$. The value 0 corresponds to the failure state and thevalue 1 corresponds to the state of perfect functionality.

Importance measures for continuous state systems have been one of the most explored area in reliability theory in recent years. The majority of continuous-state IM introduce threshold or partitioning approaches and thus reduce continuous state to binary or multi-state systems. Kim and Baxter (1987)introduced reliability importance of component iat some system stateα, $\alpha \in (0, 1)$. By using the threshold concept they reduced continuous-state system so thatstates in $[0, \alpha)$ represent the failure states and the states in $[\alpha, 1]$

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representthe operating states. Lisnianski used successive partitions of the continuous-state interval and reduced continuous-state to discrete multi-state system on which criticality measure is then applied (Lisnianski, 2001), (Lisnianski, 2002). Some of the adjustment of binary IM for continuous states can be found in (Lin, Li, & Zio, 2016) and (Liu, Si, Cui, Wang, & Sun, 2016).

4. IMPORTANCE MEASURES OF GROUPS OF COMPONENTS

One of the open issues related to the IMs is their inability to determine the importance of a group of components (Zio, 2011). In the existing literature, several approach can be found but they consider the importance of pairs or preselected groups of components. One of such IM is Joint Reliability Importance (JRI), introduced by Hong and Lie(1993) and then extended by Armstrong (1995) and Wu (2005). JRI measures joint importance of pairs of components i and j as follows:

$$JRI(i,j)=\frac{\partial^2 h(\mathbf{p})}{\partial p_i \partial p_j} \quad (12)$$

where $h(\mathbf{p})$ represents the reliability of the system and p_i, p_j reliabilities of the components i and j,respectively. In the case of statistically independent components, JRI can be calculated as:

$$JRI(i,j)=h(1_i,1_j,\mathbf{p}_{ij})+h(0_i,0_j,\mathbf{p}_{ij})-h(1_i,0_j,\mathbf{p}_{ij})-h(0_i,1_j,\mathbf{p}_{ij}) \quad (13)$$

where $\mathbf{p}_{ij}=(p_1,\dots,p_{i-1},\cdot,p_{i+1},\dots,p_{j-1},\cdot,p_{j+1},\dots,p_n)$.

Differential Importance Measures(DIM and DIMII)are new IMs that are created as an effort to apply concept of criticality to groups of components. DIM is calculated as the quotient of the total change in the system reliability caused by the change of component x_i (Borgonovo, 2007) (Borgonovo & Apostolakis, 2001):

$$DIM(x_i)=\frac{dR_{x_i}}{dR}=\frac{\frac{\partial R}{\partial x_i}dx_i}{\sum_j \frac{\partial R}{\partial x_j}dx_j} \quad (14)$$

Applied on a group of components, DIM is additive measure calculated as a sum of individual IMs from (14) (Borgonovo & Apostolakis, 2001):

$$DIM(x_i,x_j...,x_k)=DIM(x_i)+DIM(x_j)+...+DIM(x_k). \quad (15)$$

Importance measure DIMII is a combination of JRI and DIM. Complete calculation of DIMII can be found in(Zio & Podofilini, 2006). Unlike DIM, DIMII does not have additive property, i.e:

$$DIM^{II}(i,h;k,l)\neq DIM^{II}(i,h)+DIM^{II}(k,l). \quad (16)$$

Application of DIM and DIMII requires selection of the group of components in advance. This approach can be very useful if importance of some particular group of components should be investigated. However, they cannot select the group of the most importance components among all the components of the system.

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5. CONCLUSIONS

The aim of this paper was to give an overview of the most widely used importance measures. Since Birnbaum introduces the first importance measures, this area is constantly evolving and expanding to different types of systems. However, all reviewed importance measures, traditional and new one, refer to importance of individual components. Thus, after determining the numeric value values for each component individually, it is possible to extract a number of components that can be considered the most important for the reliability of the system. The most promising directions of research in IM are determining groups of critical components and analysis of components of multi-state and continuous-state systems.

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